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COSMIC STRINGS AND BARYON DECAY CATALYSIS*

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ABSTRACT

Cosmic strings, like monopoles, can catalyze proton decay. For integer charged fermions, the cross section for catalysis is not amplified, unlike in the case of monopoles. We review the catalysis processes both in the free quark and skyrmion pictures and discuss the implications for baryogenesis. We present a computation of the cross section for monopole catalyzed skyrmion decay using classical physics. We also discuss some effects which can screen catalysis processes.

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1. INTRODUCTION

It is well known[1,2] that grand unified monopoles catalyze baryon decay with a strong interaction cross section rather than with a geometric cross section. This enhancement of the cross section gives rise to some of the most stringent bounds on the number density of monopoles.[3] It is therefore of interest to investigate baryon decay catalyzed by cosmic strings.

It has been shown that - at least for integer charged fermions - there is no enhancement of the cross section for cosmic string-catalyzed baryon decay. This holds for both ordinary[4,5] and superconducting[5,6] cosmic strings. (In the case of fractionally charged fermions there will be a Bohm-Aharonov effect which leads to some amplification[7]). The analysis can be performed either using the free quark picture[4,6] or the skyrmion picture[5] for baryons.

In this review we summarize some of the issues relevant to cosmic string catalyzed baryon decay. In the following section we present some heuristic classical arguments which show why the catalysis cross section is enhanced for monopoles but not for strings. In Section 6 these classical arguments are developed further to obtain a derivation of the catalysis cross section for monopoles in the skyrmion picture[8]. In Section 3 we outline the quantum mechanical derivation of the cross section in the free quark picture, and in Section 5 we present the corresponding calculation in the skyrmion picture. In Section 7 we discuss some effects which could screen catalysis processes[9].

Our results - besides their intrinsic interest - have implications for baryogenesis[10]. As we mention in Section 4, even in the absence of any enhancement of the cross section, catalysis processes could erase a primordial baryon-to-entropy ratio. The strength of this effect, however, depends crucially on coupling constants.

We use units in which $\hbar = c = k_B = 1$. G is Newton's constant, and m_{pl} denotes the Planck mass. σ is the scale of symmetry breaking of the phase transition which produces the topological defects.

2. HEURISTIC ARGUMENTS

Baryon decay can be catalyzed by grand unified monopoles and cosmic strings because in the core of these topological defects, the gauge and scalar fields which mediate baryon number violating processes are excited. However, the baryons must be able to penetrate into the core. Without long range forces which attract the baryons to the defect, we expect that the cross section will be at most given by the geometrical cross section.

For monopoles, there is a long range force which can lead to an amplification of the cross section. Consider the wave function Ψ of the baryon. The only harmonic which does not experience an angular momentum suppression near the core is the s wave. For the s wave, the magnetic moment $\vec{\mu}$ is radial. Hence, there is a long range attractive magnetic moment-magnetic field $\vec{B}$ force

$$F(r) \sim \frac{\partial}{\partial r}(\vec{\mu} \cdot \vec{B}) \ . \tag{2.1}$$

This force leads to an amplification of the wave function of the baryon at the core radius $r_M \sim \sigma^{-1}$ of the monopole and hence to a cross section which is enhanced by A^4, where A is the ratio of the wave functions at r_M with and without the magnetic field.

It is possible to apply a similar analysis to ordinary[4)] and superconducting[6)] cosmic strings. For ordinary cosmic strings there are no long range physical fields and hence no long range forces. Therefore we do not expect any enhancement of the cross section. For superconducting cosmic strings there is a long range magnetic field - however it is proportional to $\underline{e}_\varphi$ (where φ is the angle in the plane perpendicular to the string) and hence does not yield any nonvanishing force via (2.1). Thus there will be no enhancement of the cross section even for superconducting cosmic strings. Naturally, the above discussion will miss Bohm-Aharonov type effects[7,11)].

In the following sections we shall show that the above arguments are confirmed by quantum mechanical calculations. In Section 3 we use the free quark picture (we consider the scattering of a single quark due to the background fields of the topological defect), and in Section 4 we explain how the results emerge using the skyrmion picture for baryons.

3. CATALYSIS IN THE FREE QUARK PICTURE

In this section we ignore the confining forces between the three quarks contained in a baryon and consider the one particle scattering by the background monopole or cosmic string fields.

We first derive the cross section for scattering of a quark by a monopole in the absence of any wave function amplification. We use a second quantized formalism and work to first order in perturbation theory. Hence we calculate the transition amplitude $\mathcal{A}$ between a single quark initial state

$$|i> = |q, 0> \tag{3.1}$$

and a single lepton final state

$$|f> = |\ell, 0> \tag{3.2}$$

The $'O'$ in (3.1) and (3.2) indicate that we consider states without any external gauge particles. The interaction Lagrangian is

$$\mathcal{L}_I = -ie\bar{\psi} \not{A} \psi \ , \tag{3.3}$$

with A the gauge fields mediating baryon number violating processes.

For monopoles, we can write down the expression for $\mathcal{A}$ in the absence of long range fields

$$\mathcal{A} = {}_{\infty}<f|i>_{-\infty} \sim e \int d^4x < \ell|\bar{\psi}\gamma^{\mu}\psi|q >< 0|A_{\mu}|0 > \tag{3.4}$$

up to higher order terms in coupling constants. The first expectation value is that in the Hilbert space of fermion states, the second in that of gauge particles. (3.4) can be evaluated approximately by integrating over the core, using free field wave functions:

$$\mathcal{A} \sim e\sigma^{-2} m \int dt \; e^{i(E_k - E_{k'})t} \; V^{-1/2}(E_K E_{K'})^{-1/2} \tag{3.5}$$

where m is the fermion mass. Hence, the differential cross section is

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{free}} \sim \frac{1}{T} V \int d^3k' |\mathcal{A}|^2 \sim e^2 \sigma^{-2} \left(\frac{m}{\sigma}\right)^2 \; . \tag{3.6}$$

T is the total integration time and V the cutoff volume.

The cross section with interactions is

$$\left.\frac{d\sigma}{d\Omega}\right|_{\text{int}} \sim A^4 \left.\frac{d\sigma}{d\Omega}\right|_{\text{free}} \tag{3.7}$$

where, as in Section 2, A is the ratio of the wave function including interactions to the free field wave function, evaluated at the core radius.

To determine A, we solve the Dirac equation with and without the long range gauge fields of the monopole. The Dirac equation is

$$\not{D}\,\psi - m\psi = 0 \; . \tag{3.8}$$

We choose a Weyl basis for ψ and look for stationary solutions. (3.8) then becomes

$$(H - E)\psi = 0 \tag{3.9}$$

where E is the energy of the stationary solution and

$$H = \begin{pmatrix} m & \vec{\sigma}\cdot(-i\vec{\nabla} - e\vec{A}) \\ \vec{\sigma}\cdot(-i\vec{\nabla} - e\vec{A}) & m \end{pmatrix} \tag{3.10}$$

We look for simultaneous eigenfunctions of total angular momentum J and its z

component J_z. In the Weyl basis,

$$\vec{J} = \begin{pmatrix} \vec{L} + \frac{1}{2}\vec{\sigma} & 0 \\ 0 & \vec{L} + \frac{1}{2}\vec{\sigma} \end{pmatrix} \tag{3.11}$$

The crucial point is that in the presence of a magnetic field, the orbital angular momentum obtains an extra piece

$$\vec{L} = m\vec{x} \wedge \dot{\vec{x}} - e\vec{B} \wedge \vec{x} = m\vec{x} \wedge \dot{\vec{x}} - eg\hat{x} \tag{3.12}$$

This leads to a change in the allowed angular momentum quantum numbers. Without the magnetic field, the allowed j values are half integer $j = \left(n + \frac{1}{2}\right)$, $n \epsilon Z$. In the presence of a radial magnetic field, the j values are shifted by a constant proportional to eg. In the case of monopoles we must assume the Dirac quantization condition

$$eg = \frac{n'}{2} \ , \ n' \ \epsilon \ Z \ . \tag{3.13}$$

Then, the admissible j values are $j = n, n \ \epsilon \ Z$. In particular, there is a mode for which the centrifugal potential barrier vanishes. For this mode, the wave function will be enhanced near the core compared to the modes which dominate for $\underset{\sim}{B} = 0$.

Following the methods developed in Refs. 12 and 13 we write the solutions for ψ with fixed j and m as

$$\psi_{jm}(r, \theta, \varphi) = \frac{1}{r} \begin{pmatrix} f(r) & \eta^{(1)}_{jm} & (\theta, \varphi) \\ g(r) & \eta^{(2)}_{jm} & (\theta, \varphi) \end{pmatrix} \tag{3.14}$$

where $\eta^{(i)}_{jm}$ are 2 component eigensections of J and J_z with eigenvalues $j(j+1)$ and m. In the absence of the monopole

$$g_{\frac{1}{2}}(r) \sim mr \qquad as \ r \to 0 \ , \tag{3.15}$$

whereas in the presence of the monopole

$$g_0(r) \sim \text{const} \qquad as \ r \to 0 \ . \tag{3.16}$$

Hence, taking the ratio of (3.16) and (3.15) evaluated at the monopole core radius, we obtain the amplication factor

$$A \sim \frac{g_0\left(\sigma^{-1}\right)}{g_{\frac{1}{2}}\left(\sigma^{-1}\right)} \sim \frac{\sigma}{m} \tag{3.17}$$

From (3.6) and (3.7), it then follows that the cross section for quark scattering in the background field of a monopole is

$$\frac{d\sigma}{d\Omega} \sim m^{-2} \ , \tag{3.18}$$

the well known Callan-Rubakov cross section.

In the case of an ordinary cosmic string along the z axis, we can similarly evaluate the cross section per unit length $d\sigma/(d\Omega d\ell)$ with and without long range gauge fields. The transition amplitude $\mathcal{A}$ is

$$\mathcal{A} \sim e\; m\; \sigma^{-1} \int dt\; dz\; e^{i(E_k - E_{k'})t} e^{-i(k_z - k_{z'})z} V^{-1/2} (E_k E_{k'})^{-1/2} A^2 \tag{3.19}$$

where A is the wave function enhancement factor at the core radius. Hence,

$$\frac{d\sigma}{d\Omega d\ell} \sim e^2 A^4 m \sigma^{-2}\ . \tag{3.20}$$

In the absence of purely quantum mechanical effects, we expect the factor A to be unity because of the absence of physical long range fields. The Dirac equation simplifies[14] when using the following representation of the γ matrices

$$\gamma^0 = \begin{pmatrix} \sigma_z & 0 \\ 0 & -\sigma_z \end{pmatrix},\ \gamma^1 = \begin{pmatrix} i\sigma_y & 0 \\ 0 & -i\sigma_y \end{pmatrix},\ \gamma^2 = \begin{pmatrix} -i\sigma_z & 0 \\ 0 & i\sigma_z \end{pmatrix},\ \gamma^3 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}. \tag{3.21}$$

Since the z component of the gauge field vanishes, the upper two components of ψ decouple from the lower two. The equation for the upper and lower two components is the three dimensional Dirac equation. The next step is to look for stationary solutions of the three dimensional Dirac equation with fixed momentum k and angular momentum j. If ρ and ϕ are the polar coordinates in the plane perpendicular to the string, then

$$\psi_{kj}(t,\rho,\phi) = e^{i(j\phi - \omega t)} \begin{pmatrix} \psi_+^j(k,\rho) & e^{-i\phi/2} \\ \psi_-^j(k,\rho) & e^{i\phi/2} \end{pmatrix}\ . \tag{3.22}$$

The radial functions ψ_+^j and ψ_-^j obey the Bessel equation.

Unlike for scattering by monopoles, here the admissible values do not change when we add the long range gauge fields. However, these fields do effect the index ν of the Bessel function. If the Dirac quantization condition is satisfied, then for fixed j adding the cosmic string gauge field will shift ν by an integer. Hence, the behavior of the most singular mode as $\rho \to 0$ is unchanged (although which j value this occurs for does change). Hence, there is no amplification of the free quark wave function near the core, $A \sim 1$, and there is no Callan-Rubakov enhancement of the catalysis cross section[4].

However, if the Dirac quantization condition is not satisfied, then the index ν changes by a fractional amount when adding the cosmic string field. In this case, the small ρ behavior of the most singular mode changes and there will be an enhancement of the cross section. This is a purely quantum mechanical effect of the Bohm-Aharonov type[7)].

For superconducting cosmic string the analysis is conceptually identical but technically more complicated because $A_z \neq 0$. Hence, the two upper components of ψ no longer decouple from the lower ones. We[6)] obtain a system of coupled second order differential equations for ϕ_1^+ and ϕ_2^-, the radial part of the uppermost and lowermost component of ψ. However, it can be shown that the terms which couple ϕ_1^+ and ϕ_2^- do not influence the small ρ behavior of the wavefunctions. Hence, as in the case of ordinary cosmic strings, there is no Callan-Rubakov enhancement of the cross section.

4. CATALYSIS AND BARYOGENESIS

Catalysis effects open new channels by which baryons, antibaryons and leptons can equilibrate in the very early universe. Since both initial and final states are in thermal equilibrium, no net asymmetry can be created by catalysis processes[15)]. However, a primordial baryon to entropy ratio may be erased. To check whether this will occur, we must calculate the efficiency of the process.

Let Δn be the maximal net number density of baryons converted to antibaryons by catalysis between the time t_c of the phase transition which produces strings and the present time. Δn depends on the catalysis cross section at high temperature T which, from (3.20), is

$$\frac{d\sigma}{d\Omega d\ell} \sim e^2 \, m(T)\sigma^{-2} \, . \tag{4.1}$$

Note that the finite temperature mass $m(T)$ is relevant. At temperatures $T \gg m$, $m(T) \sim T$. Δn also depends on the mean separation $\xi(t)$ of strings. Long after the strings are produced, a scaling solution with $\xi(t) \sim t$ is reached. However, at the time of formation t_c, the separation $\xi(t_c)$ is determined by microphysics[16)]. In this case $\xi(t_c)$ is the Ginsburg length

$$\xi(t_c) \sim \lambda^{-1}\sigma^{-1} \, , \tag{4.2}$$

for an abelian Higgs model with potential

$$V(\phi) = \frac{\lambda}{4}(\phi^2 - \sigma^2)^2 \, . \tag{4.3}$$

Δn is obtained by integrating dn/dt, the maximal number of baryons catalyzed per unit time and volume. dn/dt is given by

$$\frac{dn}{dt} \sim \frac{d\sigma}{d\Omega d\ell}\xi(t)\xi^{-3}(t)n_B(t)v(t) \tag{4.4}$$

where $n_B(t)$ is the number density of baryons and $v(t)$ is the mean relative speed between baryons and strings. Obviously, Δn is dominated by catalysis which takes place just after t_c. We can set $v(t) = 1$ and, using (4.1), (4.2) and $m(T_c) \sim e^{1/2}\sigma$, obtain

$$\Delta n \sim \lambda^2 e^{5/2}\sigma t_c n_B(t_c) \tag{4.5}$$

Since the baryon to entropy ratio is constant between t_c and the time t_{eq} of equal matter and radiation, it can be evaluated at t_{eq}

$$\frac{n_B(t_{eq})}{s(t_{eq})} \sim \frac{T_{eq}}{m} \tag{4.6}$$

where T_{eq} is the temperature at t_{eq} and $m \sim 1$ GeV. Evaluating T_c and dropping the e dependence, we obtain

$$\frac{\Delta n}{s} \sim \lambda^2 \frac{m_{pl}}{\sigma}\frac{T_{eq}}{m} \sim \lambda^2 10^{-5} \ . \tag{4.7}$$

Since the presently observed baryon to entropy ratio is $10^{-10} < \frac{n_B}{s} < 10^{-8}$, we conclude that, provided the coupling constant λ is sufficiently small, catalysis is too weak to destroy an initial net baryon to entropy ratio. It may seem surprising that the effect is not much smaller. It is known[17)] that monopole catalyzed baryon decay is ineffective at erasing the primordial baryon to entropy ratio, despite a large Callan-Rubakov enhancement of the cross section. However, for monopoles there is an independent bound on the number density of monopoles[18)] which gives a number density much smaller than the Kibble mechanism[16)] would predict. The bound comes from requiring that monopoles do not give an energy density in excess of closure density. For cosmic strings there is no corresponding apriori bound, since they chop themselves up efficiently into loops which in turn decay by emitting gravitational radiation. It is the large number density given by (4.2) which leads to the relatively large effect on n_B/s for cosmic strings. Note also that at high temperatures (which dominate Δn), the Callan-Rubakov enhancement factor for monopoles $(\sigma/m(T))^4$ decreases to 1. Hence[19)], we expect that (4.6) will be valid also for fractionally charged fermions.

5. COSMIC STRINGS AND SKYRMION DECAY

So far we have presented a high energy picture of baryon decay, however, since the current energies and densities in the universe are in fact low, in order to understand catalysis it is important to develop a low energy picture. One such possibility was investigated by Callan and Witten[20], who examined a skyrmion decay process in the presence of a monopole. We will examine the analogous process for a string, developing the Callan-Witten argument using the Wu-Yang picture of a monopole. This allows a ready distinction between the physical singularity of the electromagnetic fields at the core and the gauge string singularity. We examine the scattering of a skyrmion off a cosmic string, first trying the wire model for the string in order to mimic the Dirac model for the monopole, however such a picture does not permit baryon decay. We are therefore forced to consider a vortex model for the string in order to obtain catalysis in the string core. We also consider the analogous process for a superconducting string. First we use the wire model, but despite there being long range fields in this case, we again show that such a picture does not result in baryon decay. We then use a vortex model for the superconducting string and obtain catalysis in the string core. The analysis gives a heuristic explanation of the enhancement factor with monopoles, as we will show.

Let us first highlight the features of the Skyrme model relevant to the catalysis procedure. The Skyrme model[22] is a sigma model with stable soliton solutions otherwise known as skyrmions. In the case of two quark flavours (which we will be assuming here for simplicity), the pion field content is contained in an SU(2) field $U = \exp\left\{\frac{2i}{f_\pi}\vec{\tau}\cdot\vec{\pi}\right\}$, where $\vec{\tau} = (\tau_1, \tau_2, \tau_3)$ are the three generators of SU(2). The field space is thus isomorphic to S^3. Since finiteness of the energy requires that $U(\vec{x}) \to$ const. as : $\vec{x} :\to \infty$, we can think of a soliton field configuration as a map from compactified three-space ($\mathcal{R}^3 \cup \{\infty\} \cong S^3$) to the three-sphere of SU(2). Such maps may be classified according to the homotopy equivalence class to which they belong. Since $\Pi_3(S^3) \cong Z$, we may conclude that soliton field configurations are labelled uniquely by an integer value, N_B (the baryon number), which is the degree of the map. In a dynamical theory, the continuity of the fields implies that N_B is a continuous function of time and hence constant. The baryon number may also be more familiarly represented as the charge associated with the conserved baryon current

$$B_s^\mu = \frac{1}{24\pi^2}\epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}\left(U^{-1}\partial_\nu U U^{-1}\partial_\rho U U^{-1}\partial_\sigma U\right). \qquad (5.1)$$

In the presence of electromagnetism, the model must be generalised to allow for the nucleon charge and magnetic moment interaction. The Skyrme lagrangian must be invariant under the gauge transformation

$$A_\mu \to A_\mu + \partial_\mu \alpha \tag{5.2a}$$

$$U \to e^{ie\alpha(x)Q} U e^{-ie\alpha(x)Q} = e^{ie\alpha(x)\tau_3/2} U e^{-ie\alpha(x)\tau_3/2} \tag{5.2b}$$

where Q is the quark charge matrix ($Q = \frac{1}{6}I_2 + \frac{1}{2}\tau_3$). Taking into account QCD anomalies, Witten[23)] showed that the baryon current is modified:

$$B^\mu = B^\mu_s + \frac{ie}{8\pi^2}\epsilon^{\mu\nu\rho\sigma}\partial_\nu[A_\rho \mathrm{Tr}Q(U^{-1}\partial_\sigma U + \partial_\sigma U U^{-1})] \, . \tag{5.3}$$

The new A_μ dependent term is a divergence. Thus provided there are no singularities in A_μ, and that surface terms vanish, the baryon number is still integral. In terms of the topological picture presented previously, provided there are no singularities, $U(x)$ is still a map from $S^3 \to S^3$ and thus the classification of maps into equivalence classes labelled by baryon number still holds.

Callan and Witten considered a skyrmion interacting with a Dirac monopole. In a spherical coordinate basis this has a gauge potential given by

$$A_\phi = g(1 - \cos\theta) \ , \tag{5.4}$$

which is singular on the line $\theta = \pi$, however the electromagnetic flux is finite everywhere except at $r = 0$. The singularity of A_μ on $\theta = \pi$ is a gauge artifact, the Dirac string, which arises because we are trying to express the electromagnetic field tensor as the exact differential of a covector gauge field on $\mathcal{R}^3 - \{0\}$.

In order to make these intricacies more transparent, we will take an approach to the Dirac monopole which avoids Dirac strings – that due to Wu and Yang[21)]. Briefly, the singularity in A_μ can be removed if one chooses two coordinate patches for $\mathcal{R}^3 - \{0\}$, each with an associated A_μ, relating the two different 'branches' of A_μ by a gauge tranformation on the overlap. Two convenient patches are

$$(1) \quad \{0 \le \theta < \pi - \delta \ ; \ \ r > 0\} \quad ; \quad (2) \quad \{\delta < \theta \le \pi \ ; \ \ r > 0\} \tag{5.5}$$

with

$$A_{1\phi} = g(1 - \cos\theta) \qquad ; \qquad A_{2\phi} = -g(1 + \cos\theta). \tag{5.6}$$

These are related by the non-trivial gauge transformation

$$A_{2\mu} = A_{1\mu} - 2g\partial_\mu \phi \tag{5.7}$$

on the overlap. This picture now has no coordinate singularities. To include the SU(2) field, U, in this picture, we note that since the U-field is coupled to the gauge field the presence of the two branches of A_μ indicates that we must define a separate field configuration on each chart. These will then be related in the overlap by a non-trivial transformation induced by the gauge transformation (5.7) on A_μ. From (5.2) we conclude that this is

$$U_2 = e^{-i\phi\tau_3/2} U_1 e^{i\phi\tau_3/2} \ . \tag{5.8}$$

We now have a perfectly consistent, singularity free picture of the nucleon on the background field of the monopole.

Having removed the singularity problem, we see that once again the SU(2) field configuration is a map from compactified physical space into the SU(2) three-sphere. However, here we have a non-trivial transformation for U on the overlap of the two coordinate patches. Thus although we can classify the field configurations in each case according to homotopy equivalence, there is no reason to assume that in each case these classes will be the same. Indeed, the effect of the gauge transformation is to rotate the vector $\hat{\underline{\pi}}$ by an angle ϕ around the 3-axis, which will have a twisting effect on the π_1, π_2 components. Thus the presence of the monopole gauge field shuffles the members of the baryon equivalence classes. This 'shuffling' is crucial to the physical description which follows.

Solving the Klein-Gordon equation in the presence of a magnetic monopole shows that the wave functions of charged pions are suppressed by a factor of $r^{\frac{\sqrt{3}-1}{2}}$ near the core. However for uncharged particles no such suppression occurs. Thus in order for the nucleon to approach the monopole core, it must be able to deform into a pure π^0 field configuration. In order for this process to be possible, the π^0 field configuration must be able to carry baryon number. Callan and Witten found that a pure π^0 radial configuration, $U_K = \exp\{if\tau_3\}$ (where f runs from 0 at the origin to 2π at infinity), carries baryon number 1; this field configuration is called the radial kink[20]. Calculating the radial baryon flux of the kink, shows that the radial flux of baryon number into the monopole core is $\frac{\dot{f}(0,t)}{2\pi}$. Whether or not $\dot{f}(0,t)$ can be non-zero depends on the boundary conditions at the monopole core. In the case of a grand unified monopole formed during an SU(5) or SO(10) phase transition for example, it is possible for baryon non-conserving boundary conditions to be placed, and hence for $\dot{f}(0,t) \neq 0$. Thus monopoles can catalyse skyrmion decay.

We now turn to the case of a skyrmion interacting with a cosmic string. At first sight, we might expect some similarities with the monopole case, since the monopole has a semi-infinite Dirac string singularity, and we have an infinite string. However, this would be misleading; the Dirac string is a gauge singularity and can easily be removed by a more suitable description in terms of coordinate patches. In the case of a monopole we needed to define two branches of the gauge field on two different coordinate patches, related by a non-trivial gauge transformation on the overlap. The cosmic string however, has a perfectly well defined gauge field without invoking coordinate patches. Thus the gauge field for a cosmic string exhibits no singularities, the additional term in (5.3) is once more a total divergence, and baryon number is unchanged. Alternatively, if there are no gauge singularities, the equivalence classes of the soliton maps are unchanged.

In grand unified models the string width is of the order of M^{-1}, where M is the grand unified mass. Thus, to mimic the approximation of a monopole by a Dirac monopole, we take the string as a wire singularity on the symmetry axis. Away from this singularity the gauge field is given by

$$A_\mu = -\frac{1}{e}\nabla_\mu \vartheta \ , \tag{5.9}$$

in cylindrical polar coordinates $\{\rho, \vartheta, z\}$. The static Klein-Gordon equation reduces to

$$(\nabla_\mu + ieA_\mu)^2 \varphi = -\left[\frac{1}{\rho}\partial_\rho \rho \partial_\rho + \partial_z^2 + \frac{1}{\rho^2}(\partial_\theta - ieA_\vartheta)^2\right]\varphi = 0. \tag{5.10}$$

Here, rather like the monopole case, φ picks up extra "angular momentum" around the z-axis due to the presence of a non-zero A_ϑ. For the wire model, (5.10) implies that the radial part of the wave equation for the lowest angular momentum eigenstate must tend to zero as least as quickly as ρ near $\rho = 0$. Therefore, as in the monopole case, the wave functions of charged particles are suppressed near the core of the string, but those of uncharged particles need not be. Unfortunately, equation (5.3) implies that it is now impossible for a radial kink to carry baryon number, since $A_\mu = -\frac{1}{e}\nabla_\mu \phi$ is constant outside the wire.

Taking the wire approximation for a cosmic string leads to a suppression of the charged pion fields near the string. However, since a radial kink cannot carry baryon number in this case, we cannot have a deformation of the nucleon fields that would allow the skyrmion to approach the string core. Hence in the wire model of cosmic strings we do not get catalysis. Perhaps this problem is a result of approximating the string core by a line. In order to be more physically realistic, we will consider a vortex model for the string. To illustrate the salient features of skyrmion catalysis by cosmic strings it is only necessary to consider an abelian theory. Thus we consider the Nielsen-Olesen vortex[24)]. This is a vortex solution to the lagrangian

$$\mathcal{L}[\phi, A_\mu] = D_\mu \phi^\dagger D^\mu \phi - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{\lambda}{4}(\phi^\dagger \phi - \eta^2)^2, \tag{5.11}$$

where $D_\mu = \nabla_\mu + ieA_\mu$ is the usual gauge covariant derivative, and $F_{\mu\nu}$ the field strength associated with A_μ.

The Nielsen-Olesen vortex solution corresponds to an infinite, straight static string aligned with the z-axis. In this case, we can choose a gauge in which

$$\phi = \eta X(\rho) e^{i\vartheta} \quad ; \quad A^{\mu} = \frac{1}{e}[P(\rho) - 1]\nabla^{\mu}\vartheta. \tag{5.12}$$

This string has winding number one. There are no known analytic solutions for X and P, but asymptotic forms may be derived. Near the origin, these are:

$$X \propto \rho \quad ; \quad P = 1 + O(\rho^2) \qquad \text{as } \rho \to 0, \tag{5.13}$$

Using the asymptotic form for P in the Nielsen-Olesen vortex field instead of the wire form in the Klein-Gordon equation (5.10) shows that the radial equation for the lowest angular momentum eigenstate now allows $\varphi \sim$ const. as $\rho \to 0$. Thus, on the scale of the core of the string, we need not have total suppression of charged particle wave functions.

Writing the vortex field A_{μ} in spherical polar coordinates and substituting into (5.3) shows as before, although slightly less trivially, that the radial kink cannot carry baryon number. However, this is no longer critical for we can have all three pion fields approaching the core. Once the skyrmion is in contact with the core of the string, where the grand unified symmetry is essentially restored, the possibility of decay arises.

We will consider an unwinding process involving all three pion fields by making the simple ansatz that the nucleon field configuration now depends on time:

$$U_N(\bar{x}, t) = \exp[iF(r,t)\hat{\bar{x}}.\bar{\tau}]. \tag{5.14}$$

The calculation of the baryon current for this field configuration is somewhat involved[5)], the main result we need is the radial baryon current of the field configurations

$$B^r = -\frac{\dot{F}}{4\pi^2 r^2}\left[P(\cos 2F - 1) + rP'\frac{\cos^2\theta}{\sin\theta}\right] . \tag{5.15}$$

Integrating this over a sphere of radius r gives

$$\begin{aligned}\int\limits_{S^2} B^r d^2x &= \frac{-\dot{F}}{2\pi}\int\limits_0^{\pi} d\theta \sin\theta \left[(\cos 2F - 1)P(r\sin\theta) + r\frac{\cos^2\theta}{\sin\theta}P'(r\sin\theta)\right] \\ &= \frac{\dot{F}}{\pi} - \frac{\dot{F}\cos 2F}{2\pi}\int\limits_0^{\pi} d\theta \sin\theta P(r\sin\theta).\end{aligned} \tag{5.16}$$

For small r, $P(r\sin\theta) = 1 + O(r^2)$ implies that the flux of baryon number into the string core is $-\dot{F}(1-\cos 2F)/\pi$.

Thus in the presence of baryon non-conserving boundary conditions, such as we would expect in the string core where the grand unified symmetry is unbroken, the skyrmion can unwind. Since $F(0) = \pi$ and $F(\infty) = 0$ for the standard nucleon field configuration, we expect that for an unwinding process F changes from π to 0 at the core of the skyrmion. And indeed

$$\Delta N_B = \int \dot{B}_N dt \simeq \int dt \dot{F}(1 - \cos 2F)/\pi = \frac{1}{\pi}\Delta\left[F - \frac{1}{2}\sin 2F\right] = -1 \quad (5.17)$$

The residual field configurations is a topologically trivial excitation of the pion fields, and can therefore dissipate.

Thus strings can catalyse skyrmion decay. The picture however relies fundamentally on taking a vortex model for the string, *i.e.* one in which the string has a finite thickness. A model of the string with infinitesimal thickness (the wire model) gives no catalysis.

We will now comment briefly upon the generalisation to superconducting cosmic strings. Unlike their Nielsen-Olesen cousins, these have a long-range electromagnetic field, so we might expect some differences with the previous analysis. After all, one of the differences between the monopole and the Nielsen-Olesen vortex was the absence of long range interactions in the latter setup. However this is not the case as we will now show.

Similar to the cosmic string case discussed previously, we can try taking the superconducting string to be a wire singularity on the symmetry axis. The long range electromagnetic gauge field is

$$A_z(\rho) = \frac{-I}{2\pi}\log(\rho/\rho_0), \quad (5.18)$$

where ρ_0 is the radius of the string, and I is the current in the string. Imposing (5.18) for $\rho > 0$ gives a wire model for the superconducting string.

Since we now have a long range electromagnetic field, we might expect some modifications of the previous analysis. Consider first the Klein-Gordon equation. In cylindrical polar coordinates, the Klein-Gordon equation reduces to

$$(\nabla_\mu + ieA_\mu)^2\varphi = -\left[\frac{1}{\rho}\partial_\rho\rho\partial_\rho + (\partial_z - ieA_z)^2 + \frac{1}{\rho^2}\partial_\theta^2\right]\varphi = 0. \quad (5.19)$$

Thus, similar to the monopole and cosmic string cases, φ picks up extra "angular momentum" due to the presence of a non-zero A_z. When we insert the form for A_z from (5.18) into (5.19) there is no analytic solution for φ. However, it is possible to show that charged particle wave functions are suppressed near the wire, but those of uncharged particles need not be.

In order to see if the radial kink can carry baryon number we express A_μ in spherical polar coordinates. As in the previous discussions, B^μ_S (given by (5.1)) is zero for the radial kink, and the baryon number of the radial kink must be zero since there is no ϕ-component or ϕ-dependence in A_μ. Thus the previous discussion given for the ordinary cosmic string also applies to the case of superconducting cosmic strings: since the charged fields cannot approach the string core, and since a radial kink cannot carry baryon number, the nucleon cannot approach the core and unwind.

In order to obtain catalysis it seems necessary to consider a vortex model for the superconducting string. To obtain such a model, we consider the $U(1)\times U(1)'$ model of Witten[25].

The lagrangian in this case is

$$\begin{aligned}\mathcal{L} = D_\mu\phi^\dagger D^\mu\phi - \frac{1}{4}G_{\mu\nu}G^{\mu\nu} + D_\mu\sigma^\dagger D^\mu\sigma - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\\ - \left[\frac{\lambda_\phi}{4}(\phi^\dagger\phi-\eta^2)^2 + (f|\phi|^2-m^2)|\sigma|^2 + \frac{\lambda_\sigma}{4}|\sigma|^4\right],\end{aligned} \tag{5.20}$$

where ϕ and σ are complex scalar fields; $\lambda_\sigma, \lambda_\phi$ and f are coupling constants;

$$D_\mu\phi = \nabla_\mu\phi + igC_\mu\phi \quad D_\mu\sigma = \nabla_\mu\sigma + ieA_\mu\sigma,$$

C_μ and A_μ being abelian gauge fields carrying charges of g and e respectively, with $G_{\mu\nu}$ and $F_{\mu\nu}$ being the corresponding field strengths.

In analogy with the Nielsen-Olesen vortex, we consider a 'static' cylindrically symmetric superconducting string, *i.e.* one with constant current in the z-direction. (We will write this constant as ς.) This means that we can choose a gauge in which

$$\begin{aligned}\phi = R(\rho)e^{i\vartheta} \qquad C_\mu = \frac{1}{g}(P(\rho)-1)\nabla_\mu\vartheta\\ \sigma = S(\rho)e^{i\varsigma z} \qquad A_\mu = \frac{1}{e}(Q(\rho)-\varsigma)\nabla_\mu z\end{aligned} \tag{5.21}$$

The analytic expressions near the origin are

$$\begin{aligned}R \propto \rho \quad , \qquad P = 1 + O(\rho^2)\\ S = S_0 + O(\rho^2) \quad , \quad Q = \varsigma + O(\rho^2).\end{aligned} \tag{5.22}$$

As with the Nielsen-Olesen vortex, the gauge fields modify the Klein-Gordon equation. The radial equation now becomes

$$\frac{1}{\rho}\partial_\rho\rho\partial_\rho\varphi(\rho) = \left[(Q(\rho)-\varsigma)^2 + (P(\rho)-1)^2/\rho^2\right]\varphi(\rho) = O(\rho^2)\varphi(\rho) \tag{5.23}$$

which allows $\varphi(\rho) \to$const. as $\rho \to 0$. Therefore, as with the Nielsen-Olesen vortex, on the scale of the core of the string, we do not have suppression of charged particle wave functions.

In order to calculate the baryon current we require the expression of A_μ in spherical polar coordinates. From (5.3) we can see that $B^\mu = B^\mu_{NO}$, the baryon current for the ordinary (Nielsen-Olesen) cosmic string, since the gauge field has no ϕ-component or ϕ- dependence. Therefore the radial kink cannot carry baryon number. But, as with the Nielsen-Olesen vortex we will consider an unwinding of topological charge where all three pion fields approach the core of the string.

As before we use the time dependent nucleon ansatz (5.14). The calculation of the baryon current proceeds in a similar fashion to the Nielsen-Olesen case. Since $B^\mu = B^\mu_{NO}$, we get the same baryon flux as with the ordinary cosmic string, hence $\Delta N_B = -1$ as before. Thus superconducting strings catalyse baryon decay. But, since we were forced to take a vortex model, *i.e.* a string with thickness, the process proceeds on the scale of the string core.

To summarise, we have developed the argument of Callan and Witten for monopole catalysis of skyrmion decay in such a way that the effects of a topologically non-trivial gauge field are highlighted. We then explained the corresponding scenario for cosmic strings. We found that a wire model of the string was incompatible with catalysis, but that a vortex model admitted a catalysis scenario. This was also shown to be the case for superconducting strings.

These results support the following heuristic argument (which we will support in Section 6) for the enhancement factor in the case of the monopole cross-section. The monopole argument was conducted exclusively within the approximation of the Dirac monopole; the only place the concept of a grand unified monopole occurred was in invoking baryon number non-conserving boundary conditions. By contrast, a thick string or vortex model was required in order to get catalysis to occur at all in the string picture. Thus in the monopole picture, the only scale we have is the skyrmion scale on the other hand, the inescapability of the vortex model in the string case suggests that the reaction is occurring on the scale of the string radius, rather than the skyrmion radius, thus giving a grand unified cross-section.

In fact, it is possible to give a better qualitative argument for the order of magnitude of the cross-sections. For the monopole we start with the geometrical cross-section m^{-2} of the skyrmion. Catalysis then proceeds via the radial kink, and since there is no suppression of the neutral pion wave function in the presence of the monopole, the effective cross-section has the same order of magnitude as the geometrical cross-section. Similarly, for the string we start with the geometrical cross-section per unit length m^{-1}. However in this case the catalysis cannot proceed via the radial kink, and involves the full skyrmion field configuration. By examining the Klein-Gordon equation, we see that the wave functions of the charged particles involved in the catalysis process are suppressed inside the string. For distances between m^{-1} and M^{-1} $\phi \propto \rho$. But for $\rho < M^{-1}$ the relevant wave equation is (5.15) and $\phi \propto$ const. as $\rho \to 0$. To match solutions at $\rho = M$ we require that the amplitude of ϕ in the core of the string be of the order m/M. Hence, the scattering amplitude for catalysis processes will be suppressed by m/M, and the cross-sections by $(m/M)^2$, compared to the geometrical cross section. The string catalysis cross-sections will therefore be of the order $(m/M)M^{-1}$. Similar arguments apply in the case of the superconducting cosmic string.

These results support the earlier calculations involving a quark/string scattering, that is, that there is no enhancement of the baryon decay cross- sections for strings. Hence there will be no constraints on the cosmic string scenario from catalysis based on later time astrophysical processes. The arguments presented here are heuristic, however, the calculation[8)] (see Section 6) of the cross-sections using a non-relativistic spinning particle picture confirms these conclusions. The argument provides an elegant pictorial description of the skyrmion decay process. It shows clearly the difference between the monopole and string cases, and also readily obtains the superconducting string catalysis picture.

6. THE CALCULATION OF THE CATALYSIS CROSS-SECTION

We have seen in the previous sections that, in both the free quark and the skyrmion pictures, the cross-section for proton decay via cosmic strings is just the geometric cross-section, whilst that for monopoles is enhanced via the Callan-Rubakov effect. Although the Skyrme model provides additional insight into the decay process and gives an order of magnitude for the cross-section, calculation of the actual cross-section has not been possible. However, the skyrmion is a non-relativistic, spin 1/2 particle. Thus the catalysis cross-section for skyrmion decay is just the capture cross-section of a spin 1/2, non-relativistic particle by a monopole or superconducting cosmic string[8)].

The classical equations of motion of a non-relativistic spin 1/2 particle in the presence of a magnetic field are

$$m\ddot{\vec{x}} = e\dot{\vec{x}} \wedge \vec{B} + \frac{Qe}{2m}\vec{\nabla}(\vec{S}.\vec{B}) \tag{6.1}$$

$$\dot{\vec{S}} = \frac{Qe}{2m} \quad \vec{S} \wedge \vec{B} \tag{6.2}$$

where, for the Dirac monopole, the magnetic field is $\vec{B} = \vec{x}g/r^3$, m is the mass of a baryon of charge e and anomalous magnetic moment Q. The motion has conserved angular momentum

$$\vec{J} = m\vec{x} \wedge \dot{\vec{x}} + \vec{S} - eg\hat{\vec{x}} \tag{6.3}$$

which can be used to eliminate the spin $\vec{S}$ from (6.1). Using spherical polar coordinates, the ϕ and θ equations can be readily integrated. All angular dependence then cancels from the radial equation to give

$$r^3\ddot{r} = 2\Omega - h^2 - 2eg\, i \tag{6.4}$$

where Ω is a constant of integration, $h = -\frac{eg}{m}(1 + Q/2)$ and $i = Qeg/2m^2$. Equation (6.4) is simply the radial equation of a body moving in an inverse cubic central force. The separation goes to zero, *i.e.* the baryon hits the monopole, if and only if $r^3\ddot{r} < 0$.

If the baryon has speed v and impact parameter β then, after rearranging, (6.4) becomes

$$r^3\ddot{r} = v^2\beta^2 - 2i|s| \sin\nu \cos\mu$$

where μ and ν represent the spin orientation in spherical polar coordinates. Hence the hit condition becomes

$$m^2v^2\beta^2 < |s|eg\, Q\ \sin\nu\cos\mu\ .$$

Averaging over the initial orientation of spin and using the Dirac quantisation condition we obtain

$$\sigma \sim 2^{-5/2}\left(\frac{Q}{m^2v^2}\right)$$

For the proton $Q \sim 2.8$ and for the neutron $Q \sim -1.9$. This change of sign simply alters the sector of spin average which contributes, so we need only consider the magnitude of Q. Hence we obtain

$$\sigma_{\text{proton}} \sim 0.2(c/v_{\text{monopole}})^2\text{mbarns}$$

$$\sigma_{\text{neutron}} \sim 0.13(c/v_{\text{monopole}})^2\text{mbarns}$$

In a neutron star the relative velocity of the monopoles is 0.3c to give $\sigma \sim 1.3$ mbarns. This cross-section gives a bound on the monopole flux that is more stringent than that found in[3] by a factor of 3. For velocities of 0.3c the non-relativistic approximation is valid since $\gamma \sim 1.05$ and the Thomas precession term is very small. It should be noted that our calculation gives $\sigma v^2 \sim$ constant. Previous estimates of the monopole flux have used $\sigma v \sim$ constant, which comes from a model calculation of Rubakov[2], where the capture cross-section was neglected[26].

Rather than using the magnetic field of a magnetic monopole in (6.1) we can use the field of a superconducting cosmic string

$$B = \frac{I}{2\pi r} \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{6.5}$$

In this case we use cylindrical polar coordinates and take the string to run along the z axis. The axial symmetry of the string yields conservation of the z component of the total angular momentum only, thus it is not possible to eliminate the spin from the equations of motion as in the monopole case. The conserved quantity, which we denote by J_z, is

$$d/dt \quad (mr^2\dot{\theta} + S_z) = 0 \tag{6.6}$$

We also have energy conservation

$$E = \frac{1}{2}m\dot{\vec{x}}^2 - \frac{Qe}{2m}(\vec{S}.\vec{B}) \tag{6.7}$$

Using (6.6) and noting that we can integrate the z component of the equation of motion to give an equation for $\dot{z}$ we obtain

$$\frac{2E}{m} - \dot{r}^2 = \frac{(J_z - S_z)^2}{m^2r^2} + \left(\frac{eI}{2\pi m}\right)^2 \log^2\left(\frac{r}{r_0}\right) - \frac{Qe}{2m^2}\frac{I}{\pi r}S_\theta \tag{6.8}$$

where r_0 is the distance at which $\dot{z} = 0$.

The first term will dominate at small enough r, thus the only particles able to reach the core are those with $J_z - S_z = 0$. This requires the initial conditions $\dot{\theta} = 0$. However, if we consider a classical distribution of initial spins and orbital angular momenta, there are no particles satisfying these stringent conditions. Thus the cross-section is zero in the limit of zero string width, as we found earlier.

We can substitute in the various constants into (6.8) and multiply both sides by r^2 so that the right hand side is written in terms of rI. The resulting equation is then displayed graphically in Fig.(1), where a line of constant energy is also drawn.

For the initial conditions $r = r_0, \dot{r} = \dot{r} = 0, \dot{z} = 0$ the particle starts at the bottom of a sharp dip in the potential, the height of the barrier being approximately $400I^2m$. To surmount this barrier the particle requires an initial speed $\dot{r}_0 \sim 10Im/s$. Thus the maximum current in the string for which a nonrelativistic particle can reach the top of the barrier is $I \sim 10^7$ amps. This is several orders

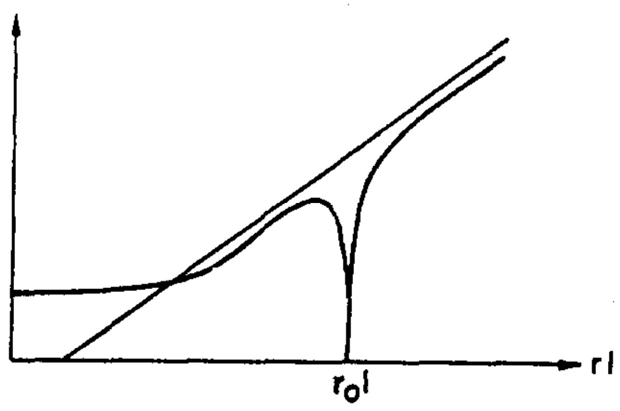

FIG. (1): General form of equation (6.8) plotted on a log-log scale. The curve represents the effective potential and the straight line is of constant energy.

of magnitude less than estimates of the maximal string current[25,27]. Further, from Fig. (1) we can estimate the maximum current in a string that allows a non-relativistic particle to penetrate until it is stopped by the centrifugal barrier. This yields the constraint $I < 10^6(\dot{r}_0/c)$ amps. Hence, for the maximal string current the non-relativistic approximation breaks down. This suggests that proton decay via superconducting cosmic strings only occurs at very high velocities.

7. SCREENING EFFECTS

So far, we have seen that cosmic strings can catalyze baryon decay, albeit with a grand unified cross section. However, we have not considered any effects which may screen the interaction. One origin of screening is the nontrivial spatial geometry of a cosmic string.

It has been shown[28] that space around an infinitely long straight string has the form of a snub-nosed cone; that is, at the core of the string space is flat while asymptotically it is conical. The deficit angle of the cone is $8\pi G\mu$, where μ is the mass per unit length of the string, and typically $G\mu \sim 10^{-6}$. Scattering of bosons and fermions on a cone has been considered in Ref. 29.

Due to the difficulty in working with the fully coupled matter and gravitational equations, most analyses of the Callan-Rubakov effect ignore the gravitational effects of the string. However, Smith[9] and Linet[30] have shown that a test charge in a conical space experiences an electrostatic self-force which is repulsive and scales as $1/r$, where r is the distance from the apex of the cone. In this section, we investigate the consequences of this self-force for catalysis. We find a potential barrier of height about 10^7 GeV.

To understand the origin of the repulsive self-force, consider as a simple case the self-potential of a test charge in a conical space with deficit angle π. In the wedge representation, *i.e.* flattening out the cone, the potential problem is that of a point charge in the upper half plane with Neumann boundary conditions.

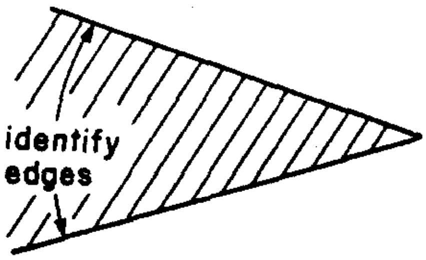

FIG. (2a): The general wedge representation. Shading indicates wedge not convered by the core.

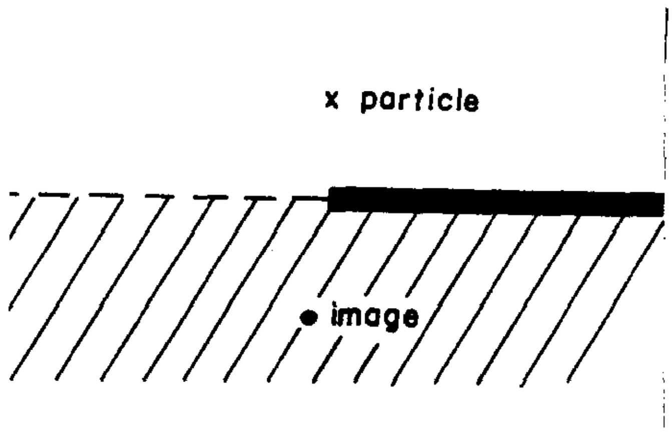

FIG. (2b): Deficit angle = π. Identify the dashed and bold lines.

To see this, cut the cone opposite to the charge (Figure 2). The apex of the cone becomes the origin, the test charge lies on the y axis and the cut edge of the cone becomes the positive and negative parts of the x axis. Since the two sides of the cut are to be identified, the potential must satisfy $\phi(x) = \phi(-x)$ at $y = 0$. Furthermore, by rotational symmetry it follows that $\partial\phi/\partial y\big|_{y=0} = 0$. Thus we have Neumann boundary conditions. The potential is now easily found by introducing an image charge of the same magnitude and sign at the site of the test charge reflected about the x axis. Hence, there is a repulsive potential proportional to $1/r$.

For the singular cone, Smith[9)] calculated the self energy of a particle of charge e. The resulting self force is (with $p = 1 + 4G\mu$)

$$\begin{gathered} \underline{F} = \frac{k(p)e^2}{2r^2}\hat{\underline{r}} \\ \text{with } k(p) \simeq \frac{\pi}{2}G\mu \ . \end{gathered} \tag{7.1}$$

The space-time of a cosmic string does not have a singularity at the origin. A more realistic space-time structure is that of a snub-nosed cone[28)] which is a consequence of the vortex model[20)]. Using the symmetries of the problem, we can write the metric in the form

$$ds^2 = e^{\gamma}(dt^2 - dr^2 - dz^2) - \alpha^2 e^{-\gamma} d\theta^2 \tag{7.2}$$

with induced Laplacian

$$\nabla^2 = e^{-\gamma}\left(\frac{\partial^2}{\partial r^2} + \left(\frac{\alpha'}{\alpha} - \frac{\gamma'}{2}\right)\frac{\partial}{\partial r} + \frac{e^{2\gamma}}{\alpha^2}\frac{\partial^2}{\partial^2\theta} + \frac{\partial^2}{\partial^2 z}\right),$$

(7.3)

where $'$ denotes differentiation with respect to r. Inserting the above ansatz into the Einstein equations leads to the following diffential equations for the functions $\alpha(r,t)$ and $\gamma(r,t)$:

$$\begin{aligned} \tilde{\alpha}'' &= -\epsilon\tilde{\alpha}e^{\gamma}(E - P_\rho) \\ (\tilde{\alpha}\gamma')' &= \epsilon\tilde{\alpha}e^{\gamma}(P_\rho + P_\theta) \\ \tilde{\alpha}\gamma &= \epsilon\tilde{\alpha}e^{\gamma}P_\rho + \frac{\tilde{\alpha}\gamma'^2}{4} \end{aligned} \tag{7.4}$$

It is convenient to write these equations in terms of dimensionless variables $\tilde{\alpha} = \alpha/r_s, \rho = r/r_s$ (where r_s is the radius of the string) and $\epsilon = 8\pi G\mu$. E, P_ρ and P_θ are also dimensionless and can be obtained from the corresponding components of $T^\mu{}_\nu$ by dividing by $\lambda\mu^2$.

In order to determine E, P_ρ and P_θ it is sufficient to consider the flat space matter field equations. The fully coupled system (*i.e.* matter equations coupled to the snub-nosed cone dynamical background) has been considered[31)] and it was shown that the flat space solutions of the matter field equations are a good approximation to those obtained from the fully coupled system.

Proceeding along the lines of Ref. 9, we can expand the self potential into eigenfunctions of L_z. The m'th harmonic satisfies the following radial differential equation

$$\phi_m'' + \left(\frac{\alpha'}{\alpha} - \frac{\gamma'}{2}\right)\phi_m' - \left(\frac{e^{2\gamma}}{\alpha^2}m^2 + k^2\right)\phi_m = 0 \tag{7.5}$$

In the case of the singular cone, $\alpha = r/p$ and (7.5) is a modified Bessel equation. For the snub-nosed cone, α/r and γ are no longer constant. There exists no exact analytical solution for α and γ. However, for small r we have[28)]

$$\alpha \sim r \qquad \text{and} \quad \gamma(0) = \gamma'(0) = 0 \tag{7.6}$$

whereas for large r $(r \geq (2-3)r_s)$

$$\alpha = ar + b \qquad \text{and} \quad \gamma = \text{const} \tag{7.7}$$

To first order in $\epsilon, a = 1 - 0(\epsilon), b = 0(\epsilon)$ and $\gamma = 0(\epsilon)$. Inserting (7.6) and (7.7) into (7.5) we find that the right hand side becomes $0(\nu')$, where $\nu = me^{\gamma}r/\alpha$. For slowly evolving deficit angles $\nu' \sim 0$. Thus, in this approximation (7.5) is essentially a modified Bessel equation with r dependent constants. In this case the method of Smith[9)] can be used, and leads to a similar result, but with an evolving value of p. At $r = 0, p = 1$ and there is no self-force. This is expected since the space-time is flat at the centre of the string. For $r > 2r_s$, p is fixed at its large r value of $1 + 4\pi G\mu$, and we obtain the $1/r$ potential of Smith for distances 2-3 times r_s. Closer than this the deficit angle diminishes and the potential drops to zero. The height of the screening potential can be estimated from the value of the singular string potential at $r = 2r_s$. For $G\mu = 10^{-6}$ the height is about 10^7 GeV.

To conclude, we have found that taking into account the structure of space-time around a cosmic string leads to a potential barrier of height about 10^7 GeV. Classically, this would restrict catalysis to a high energy process. Quantum mechanically, there is tunnelling through this barrier, as discussed in Ref. 32.

8. CONCLUSIONS

In this review we have seen that, in both the free quark and skyrmion pictures, cosmic strings and superconducting strings catalyse baryon decay with a grand unified cross-section. This is in contrast to the monopole case where the cross-section is enhanced via the Callan-Rubakov effect. We have seen that this difference can be traced to the presence of long-range, attractive forces in the monopole case which cause the wavefunction to be enhanced in the monopole core. In contrast, there are no attractive forces in the cosmic string case, and thus no enhancement of the wavefunction in the string core. Thus the cross-section is just the geometric cross-section. This is the case for integer flux. For fractional flux there is an enhancement of the wavefunction in the string core due to the Aharanov-Bohm effect.

Despite the small cross-section, cosmic string catalysis can have physical consequences in the early Universe. Near the phase transition the number density of strings is very large and can erase a substantial fraction of a pre-existing baryon asymmetry. Whilst the cross-section is enhanced for non-integer flux we have briefly discussed how the amplification factor is damped at finite temperature, in a similar manner to that of the monopole case. Hence, it is unlikely to have implications for baryogenesis over and above that already discussed for integer flux.

In the Skyrme model we have actually been able to estimate the cross-section for monopole catalysis. Since the skyrmion is a non-relativistic, spin 1/2 particle the cross-section is just the capture cross-section, found by solving the classical equations of motion. For the superconducting cosmic string we found a potential barrier that seems to indicate that catalysis is a high velocity process.

Finally we have considered effects that might screen the cosmic string catalysis cross-section. By taking into account the non-trivial space-time structure around a cosmic string we have shown that charged particles encounter a potential barrier of height 10^7 GeV. Classically this limits string catalysis to a high energy process near the phase-transition, though quantum mechanically there is the possibility of tunnelling.

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